

THE HIGH SCHOOL FINALS



The Finals will be conducted in rounds. One at a time, each remaining contestant will have **two and a half minutes** to compute an indefinite integral. If answered correctly, the contestant remains in the competition. Once every remaining contestant has attempted one problem, a round is completed. If during any round, all contestants are unable to complete a problem correctly, all contestants will remain in the competition for another round.

The last person remaining wins an additional \$75 and will be crowned the **Integration Champion**. The winner wins an additional

INTEGRAL #1

**READY,
GET SET,...**

2:30



INTEGRAL #1

INTEGRAL #1

$$\int \sin x \sqrt{\cos x} \, dx$$

$$= - \int \sqrt{u} \, du \quad u = \cos x; \quad du = -\sin x \, dx$$

$$= -\frac{2u^{3/2}}{3} + C$$

$$= -\frac{2 \cos^{3/2} x}{3} + C$$

INTEGRAL #2

**READY,
GET SET,...**

2:30

INTEGRAL #2

$$\int \frac{1}{\sqrt{x} (2012 + \sqrt{x})^5} dx$$

INTEGRAL #2

$$\int \frac{1}{\sqrt{x} (2012 + \sqrt{x})^5} dx$$

$$= 2 \int \frac{1}{u^5} du \quad \left[u = 2012 + \sqrt{x}; \quad du = \frac{1}{2\sqrt{x}} dx \right]$$

$$= 2 \left(-\frac{1}{4u^4} \right) + C$$

$$= -\frac{1}{2 (2012 + \sqrt{x})^4} + C$$

INTEGRAL #3

**READY,
GET SET,...**

2:30

INTEGRAL #3

$$\int x (2x^3 + 1)^2 dx$$

INTEGRAL #3

$$\begin{aligned} & \int x (2x^3 + 1)^2 dx \\ &= \int x (4x^6 + 4x^3 + 1) dx \\ &= \int (4x^7 + 4x^4 + x) dx \\ &= \frac{x^8}{2} + \frac{4x^5}{5} + \frac{x^2}{2} + C \end{aligned}$$

INTEGRAL #4

**READY,
GET SET,...**

2:30

INTEGRAL #4

$$\int \frac{x + 1}{(x^2 + 2x + 2012)^9}$$

INTEGRAL #4

$$\int \frac{x + 1}{(x^2 + 2x + 2012)^9}$$

$$= \frac{1}{2} \int \frac{du}{u^9} \quad u = x^2 + 2x + 2012; \quad dx = 2(x + 1) dx$$

$$= \frac{1}{2} \left(-\frac{1}{8u^8} \right) + C$$

$$= -\frac{1}{16(x^2 + 2x + 2012)^8} + C$$

INTEGRAL #5

**READY,
GET SET,...**

2:30

INTEGRAL #5

$$\int \frac{x}{7x^2 + 7} dx$$

INTEGRAL #6

**READY,
GET SET,...**

2:30

INTEGRAL #6

$$\int \frac{1}{x^3} \sqrt{1 + \frac{1}{x^2}} dx$$

INTEGRAL #6

$$\int \frac{1}{x^3} \sqrt{1 + \frac{1}{x^2}} dx$$

$$= -\frac{1}{2} \int u^{1/3} du \quad \left[u = 1 + \frac{1}{x^2}; \quad du = -\frac{2}{x^3} dx \right]$$

$$= -\frac{1}{2} \cdot \frac{3u^{4/3}}{4} + C$$

$$= -\frac{3}{8}$$



INTEGRAL #7

INTEGRAL #7

$$\int \frac{\sin \sqrt{x}}{\sqrt{x}} dx$$

INTEGRAL #7

$$\int \frac{\sin \sqrt{x}}{\sqrt{x}} dx$$

$$= 2 \int \sin u du \quad \left[u = \sqrt{x}; \quad du = \frac{1}{2\sqrt{x}} dx \right]$$

$$= -2 \cos u$$



INTEGRAL #8

$$\int \frac{\sin x - \cos x}{\sqrt{\sin x + \cos x}} dx$$

INTEGRAL #8

$$\int \frac{\sin x - \cos x}{\sqrt{\sin x + \cos x}} dx$$

$$= - \int \frac{du}{\sqrt{u}}$$

$$u = \sin x + \cos x;$$

$$du$$

z z

INTEGRAL #9

0

**READY,
GET SET,...**

2:30

INTEGRAL #9

$$\int \frac{x^5}{x^3 + 1} dx$$

INTEGRAL #9

$$\int x^5 \sqrt{x^3 + 1} dx$$

one of many possible subs: $u = x^3 + 1$; $du = 3x^2 dx$

$$= \frac{1}{3} \int (u - 1) \sqrt{u} du = \frac{1}{3} \int (u^{3/2} - u^{1/2}) du$$

$$= \frac{1}{3} \left(\frac{2(x^3 + 1)^{5/2}}{5} - \frac{2(x^3 + 1)^{3/2}}{3} \right) + C$$

INTEGRAL #10

**READY,
GET SET,...**

2:30

INTEGRAL #10

$$\int (\sin x + \cos x)^2 dx$$



INTEGRAL #11

**READY,
GET SET,...**

2:30

INTEGRAL #11

$$\int \frac{1}{x\sqrt{x}} \left(2 + \frac{1}{\sqrt{x}} \right)^4 dx$$



INTEGRAL #11

$$\int \frac{1}{x\sqrt{x}} \left(2 + \frac{1}{\sqrt{x}} \right)^4 dx$$

$$= -2 \int u^4 du \quad \left[u = 2 + \frac{1}{\sqrt{x}}; \quad du = -\frac{1}{x\sqrt{x}} \right]$$

$$= \frac{-2u^5}{5} + C$$

$$= -\frac{2}{5} \left(2 + \frac{1}{\sqrt{x}} \right)^5 + C$$

INTEGRAL #12

**READY,
GET SET,...**

2:30

INTEGRAL #12

$$\int \frac{\sin 2x}{\cos^3 x} dx$$

INTEGRAL #12

$$\int \frac{\sin 2x}{\cos^3 x} dx$$

$$= \int \frac{2 \sin x \cos x}{\cos^3 x} dx = 2 \int \frac{\sin x}{\cos x} \cdot \frac{1}{\cos x} dx$$

$$= 2 \int \sec x \tan$$